NASA SP-5939 (01)

TECHNOLOGY UTILIZATION

THEORETICAL AND APPLIED MATHEMATICS

A COMPILATION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Foreword

The National Aeronautics and Space Administration and the Atomic Energy Commission have established a Technology Utilization Program for the dissemination of information on technical developments which have potential utility outside the aerospace community. By encouraging multiple application of the results of their research and development, NASA and AEC earn for the public an increased return on the investment in aerospace research and development programs.

This publication, part of a series to provide such technical information, is intended for mathematicians, engineers, physicists, and other persons with training, knowledge, and interest in mathematics. A small part of the large field of theoretical and applied mathematics is covered. Section 1 primarily concerns mathematical applications for the resolution of problems encountered in numerous industries. Section 2 concerns computer programs which can be applied to resolve computational problems swiftly and accurately.

The usefulness of this material is indicated by its applications to aircraft, automobiles, beams, cylinders, structural elements, oil fields, geophysics, missiles, plates, solid rocket propellants, mobile nuclear reactors, rods, solutes, solvents, space vehicles, spheres, pressure vessels, and ultrasonics.

Additional technical information on individual devices and techniques can be requested by circling the appropriate number on the Reader's Service Card included in this compilation.

Unless otherwise stated, NASA and AEC contemplate no patent action on the technology described.

We appreciate comments by readers and welcome hearing about the relevance and utility of the information in this compilation.

Ronald J. Philips, *Director*
Technology Utilization Office
National Aeronautics and Space Administration

Contents

SECTION 2. Computer Programs for Industrial Applications

Section 1. Mathematical Applications

CURVE FIT TECHNIQUE WITH FIRST DERIVATIVE CONTINUITY USING A MINIMUM NUMBER OF ANALYTICAL EXPRESSIONS

This curve fit technique is a chain system of connected segments or arcs of the total curve, where this curve is single valued at all points with continuous first derivatives at these points. The analytic expression, $y=A/(1-Bx)+Cx^2+Dx+E$, closely represents each segment.

The coefficients A, B, C, D, and E are determined by three points on each arc or segment and the two slopes of the first and third points, where these two points represent the ends of the segment or analytical expression. Determinations of the coefficients are given in a detailed development of the equations.

The technique requires less analytical expressions with more accuracy than other currently used techniques; it also offers a simple, more direct method of solution of the coefficients from the curve fit for three points and the adjacent two slopes of the curve, and elimination of the complications of wave forms, or excursions, of the curve between adjacent sets of points.

Source: T. M. Dannback of
The Boeing Co.
under contract to
Marshall Space Flight Center
(MFS-14735)

Circle 1 on Reader's Service Card.

COMPUTATIONAL PROCEDURE FOR A MINIMUM–MAXIMUM POLYNOMIAL CURVE FIT ROUTINE

Some form of least-squares curve fit is commonly employed to fit a span of data or to approximate a mathematical function for use in a computer. If the maximum error resulting from the approximation is of interest, however, this procedure is not optimum. The maximum error may be reduced 10 to 20% by iteration on the curve fit.

A mathematical procedure for improving fitting between a set of data points and a polynomial uses a minimum–maximum fit, i.e., one in which all the maximum errors are of the same size. This procedure uses a least-squares iteration; the dependent variable is the deviation of each local maximum (including end points) from the average value of the local maxima; the independent variable is the array of polynomial coefficients.

An initial least-squares fit is assumed to be available. The error curve is computed. For a polynomial approximation of degree n, an error curve with n + 1 values of zero and n + 2 local maxima (including the endpoints) normally results. The "average" maximum error (either the arithmetic average of magnitudes or root-mean-square) and the deviations from this average for each local maximum are determined. A correction to the polynomial coefficients involving minimization of the extremes of the error curve is computed by a weighted least-squares method. The above steps from computation of the error curve are repeated until the deviations of the errors from the average are within some specified tolerance.

The program is designed to remove extraneous local maxima and drive the curve fit to the desired solution. Isolated cases in which the endpoint is not a local maximum may also be treated. The logical flow to delete extraneous local maxima is believed to be novel.

A method is provided for the best polynomial approximation in the sense that the error curve has the lowest maximum error; this is obtained at a loss of 1 to 2% greater root-mean-square

error. The maximum error is reduced 10 to 20%. There is a potential wide use for this method in computer science to evaluate complex geometric surfaces, such as the design of automobile bodies and aircraft fuselage and wing configuration.

Source: J. B. Clifford, Jr. of
TRW Systems Group
under contract to
Manned Spacecraft Center
(MSC-13257)

Circle 2 on Reader's Service Card.

RATIONAL CHEBYSHEV APPROXIMATIONS FOR FERMI–DIRAC INTEGRALS OF ORDERS −1/2, 1/2, and 3/2

The complete Fermi–Dirac integrals are usually defined by $F_k(x) = \int_0^\infty \frac{t^k dt}{e^{(t-x)}+1}$, $k > -1$. These integrals appear in a variety of applications subject to Fermi–Dirac statistics, for example, in the theory of semiconductors. The most frequently used functions are those for which the order k is either an integer or a half-integer. Function values are quite difficult to compute for k a half-integer and x positive.

Previously, interpolations from data compiled in tabular form were used to generate a compatible pair of Chebyshev approximations for k = 1/2. This allowed easy computation of $F_{1/2}(x)$ with a maximal relative error less than 5×10^{-4}.

Portions of the $L\infty$ Walsh arrays of rational Chebyshev approximations for k = −1/2, 1/2, and 3/2 are now available in tables. Maximal relative errors vary with the function and interval considered but generally range down to 10^{-9} or less.

Source: W. J. Cody and H. C. Thacher, Jr.
Argonne National Laboratory
(ARG-10454)

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TABLE OF THE SOLUTIONS OF a TAN (πx) = −b TAN (aπx)

A reference table has been compiled which gives the first ten positive values of x satisfying the equation of a tan (πx) = −b tan (aπx) to four places for several values of a and b in the range $0.001 \leq a \leq 1.0$ and $0.001 \leq b \leq 1000$.

These solutions are used in the series solution of a problem involving transient diffusion in a composite slab. An eigenfunction technique is employed in this problem with the eigenvalues given by $\tan \lambda_k = -M/a \tan \alpha \lambda_k$, for k = 0,1,2,... .

The transient response of two slabs of finite thickness in contact at a plane interface and initially containing nonequilibrium amounts of a transferable solute is of practical importance in the analysis of porous media in oil fields, and in the approach to equilibrium of a solute distributing between two immiscible liquid layers.

Source: P. Concus and D. R. Olander
Lawrence Radiation Laboratory
(LRL-10007)

Circle 4 on Reader's Service Card.

REDUNDANT FORCE PROGRAM TO SOLVE VIBRATION RESPONSE PROBLEMS

Missiles and space vehicles during static firing and flight experience dynamic vibrations which are usually sufficiently severe to degrade the structural integrity. An analytical method that reliably predicts the response to a random environment has been developed and is available.

This method is used in the analysis of redundant structures to obtain the dynamic response for

sinusoidal and random excitation. Random motion of the base and multiple random loadings are considered. For example, the response to random motion of the base, an acoustic field, and both excitations simultaneously may be determined.

Equations are developed for computing the response using generalized parameters. Cross power spectral densities are used to account for the correlation of the multiple random inputs. A lumped parameter idealization is employed in the method and structural influence coefficients, the natural frequencies and modes, and the dynamic response of the structure are resolved.

For the purpose of analysis, the required information consists of mass, damping, and stiffness matrices which may be obtained in a large variety of methods. If facilities are available, these parameters may be obtained experimentally.

Once the transfer functions have been computed, the choice of the type of excitation to which the structure is subjected and the type of response to be computed is nearly unrestricted. The results include the accelerations, motions, and structural component loads for the system being analyzed.

Versatility is the main advantage of this method. Its efficiency is demonstrated when used in conjunction with digital computers. The experienced user has the capability of increased accuracy by the use of more normal modes or increased efficiency by using the most significant normal modes when computing the total response.

Source: C. M. Fuller and D. N. Roudebush of
McDonnell-Douglas Corp.
under contract to
Marshall Space Flight Center
(MFS-12319)

Circle 5 on Reader's Service Card.

A WAVE TRANSMISSION METHOD OF ANALYSIS OF A TIMOSHENKO BEAM

A better insight is gained into the problem of transverse bending of a beam having secondary dynamic effects of rotary inertia and shear compliance by the wave transmission approach. It provides a physically realistic representation of the structure and more accurate solutions than the standard-lumped model achieved through modal techniques. This is true since structural parameters are distributed and not lumped in actual conditions.

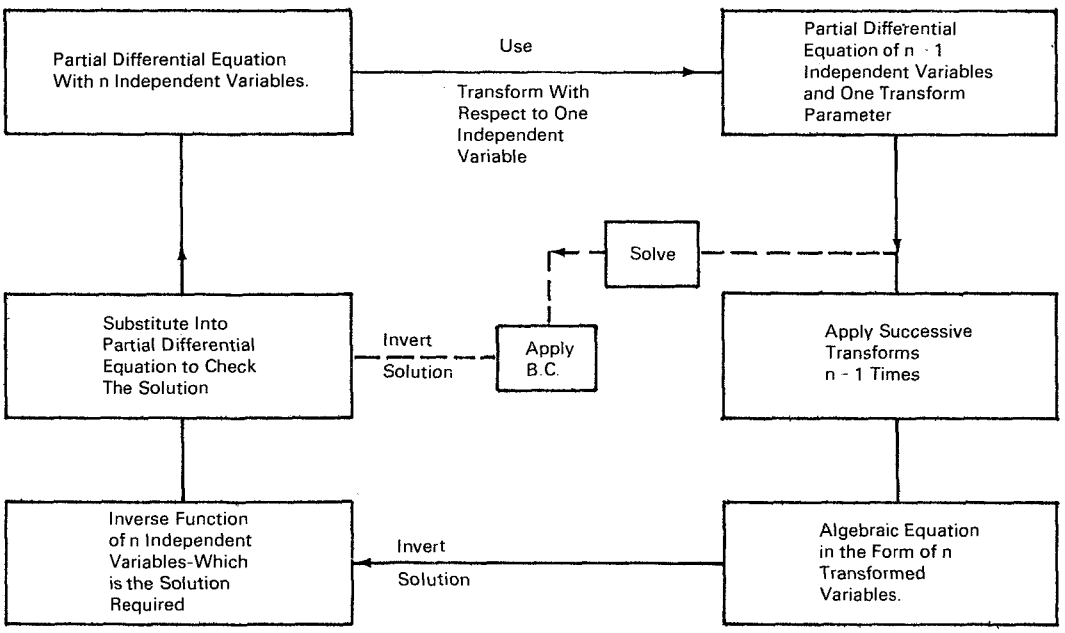
The distributed approach results in an overall transmission matrix relating a set of system variables at one end of a beam to those at the other end, or may represent segments of the structure with variation in beam parameters. Also, two end-effect matrices and two propagation matrices are obtained through the solution of the Timoshenko beam equation in terms of the distributed system concepts of propagation and reflection, and through a transformation technique relating the local state variables to the characteristic variables of the beam.

Source: W. C. Nowak of
McDonnell Douglas Corp.
under contract to
Marshall Space Flight Center
(MFS-12808)

Circle 6 on Reader's Service Card.

THE USE OF FOURIER TRANSFORMS IN THE SOLUTION OF BEAM AND PLATE PROBLEMS

Fourier transforms can be used advantageously in the solution of static and dynamic problems of structural elements. The ordinary or the partial differential equation which describes the boundary value and/or initial value problem together with its prescribed boundary and initial conditions can be simplified by means of an integral transform and solved exactly, in closed form. This technique reduces a partial differential equation with n independent variables to (n − 1) independent variables and one transform parameter if the transform of the original differential equation is taken



with reference to one variable. By successive applications of the transform (n − 1) times with respect to the remaining (n − 1) transform variables, one obtains an ordinary differential equation with (n) transform parameters. Obtain the solution of the ordinary differential equation and use the transforms of the boundary conditions and then obtain the inverse. In the general transform approach (which includes Laplace, Mellin, Henkel and complex Fourier) the inverse is an integral and is not easy to solve. However, in finite Fourier Transforms, the inverse is a summation. However, sometimes it is more convenient to take n transforms with respect to n independent variables. The various steps in the solution are shown in detail in the figure.

Details of the general principles and some applications emphasizing Fourier finite sine transforms and Fourier finite cosine transforms only are available.

Source: C. L. Amba-Rao of
Wyle Laboratories
under contract to
Marshall Space Flight Center
(MFS-12555)

Circle 7 on Reader's Service Card.

DYNAMIC BEHAVIOR OF CIRCULAR PLATES AND BEAMS WITH INTERNAL AND EXTERNAL DAMPING

Nuclear reactors give rise to numerous problems involving the dynamic behavior of elastic systems. Some of the more prominent concerns involve rapid startup or shutdown of reactors; control-rod scram; rapid opening and closing of pressure valves; and fluid-structure interaction in the core and heat transfer system. In the case of mobile reactor and space vehicles, where weight is at a premium, complete system failure can result if vibration problems receive inadequate attention. The development and use of some new versions of finite integral transforms in solving structural dynamic problems associated with nuclear reactor design are straightforward.

A report is available which considers the dynamic response under various boundary conditions of circular plates including a rigidly clamped plate, simply supported plate, and clamped plate with an edge moment; the dynamic response of a cantilevered beam; and the dynamic interaction of a circular plate and cantilevered beam.

The concept of mechanical impedance provides

a means for determining when it is possible to use an isolated elastic system in reactor design. For a single-degree-of-freedom system, the definition of impedance is the ratio of applied force to velocity for sinusoidal excitation.

Use of the finite transform technique enables the solution of dynamic problems to be expressed in terms of normal modes. The solution appears as the sum of an infinite number of single-degree-of-freedom systems and facilitates physical interpretation and calculation. For lightly damped systems, the first-mode term approximates the solution to within 2% accuracy.

The main advantage of the finite transform approach is that it handles static, forced vibration, aperiodic loads and time-dependent boundary conditions in a direct and simple manner, while avoiding the complex integration usually associated with the Laplace transform technique.

Use of the impedance concept can play an important role in the analysis and synthesis of dynamic design problems and can simplify the solution of coupled elastic systems.

The displacement (stress) is limited solely by the amount of damping in lightly damped systems. Inclusion of internal damping leads to the result that the stress is dependent upon strain rate as well as strain, even for systems remaining within the elastic limit.

Source: G. Cinelli
Argonne National Laboratory
(ARG-10179)

Circle 8 on Reader's Service Card.

DYNAMIC VIBRATIONS OF THICK-WALLED ELASTIC ANISOTROPIC CYLINDERS AND SPHERES WITH INTERNAL DAMPING

The dynamic response of thick elastic anisotropic cylinders and spheres is encountered in problems involving such diverse fields as pressure vessels, solid rocket propellants, geophysics, and ultrasonics. New integral transforms are used in determining the transient displacement and stresses of material with transverse curvilinear isotropy and internal viscous damping for plane-strain motion of an infinitely long circular-cylindrical shell, torsional motion of a finite circular-cylindrical shell, and radially symmetric motion of a spherical shell. Loads on the surfaces are taken as arbitrary functions of space (torsional case only) and time.

The solution for the isotropic body is found by specifying certain values for the elastic constants. Cases of the solid body, thin shell, and cylindrical cavity in an infinite medium are obtained as limiting cases of the thick shell solution. An extended Weber transform is introduced which permits solution of infinite media problems in a more direct manner.

By specializing the surface tractions to standard forms such as an impulse, step function, sinusoid, etc., the free, harmonic, and static motions are recovered. Resonance and mechanical impedance are derived and studied. Radially symmetric motion of a sphere is shown to be analogous to the plane-strain motion of the cylinder.

Source: G. Cinelli
Argonne National Laboratory
(ARG-10178)

Circle 9 on Reader's Service Card.

PREDICTION METHODS FOR FRICTION COEFFICIENTS FOR GASES

Mathematical formulas are available for correlating both laminar and turbulent friction coefficients for gases, with large variations in the physical properties, flowing through smooth tubes. In recent years, the generally accepted relations used to predict friction coefficients for laminar and turbulent flow have undergone extensive experimental investigation. Observed coefficients were found to be as much as three times the predicted value.

Since 1967, empirical relations have been in use for laminar and turbulent flow wherein the

friction coefficient based on bulk temperature is a function of modified Reynolds number based on surface temperature.

For modified surface Reynolds numbers less than 3000 (corresponding to laminar flow), local and average friction coefficients can be predicted within ±20 percent for $1 < T_s/T_b < 4.1$ by the expression:

$$\frac{f}{2} = \frac{8}{Re_s}$$

where:

f = friction coefficient
T_s = surface temperature
T_b = bulk temperature
Re_s = surface Reynolds number

For modified surface Reynolds number of 3000 and greater (corresponding to turbulent flow), local and average friction coefficients for both cooling and heating ($0.35 < T_s/T_b < 7.4$) can be predicted within ±10 percent by the expression:

$$\frac{f}{2} = \left(0.0007 + \frac{0.0625}{Re_s^{0.32}}\right)\left(\frac{T_s}{T_b}\right)^{0.5}$$

These relations have been used to correlate friction coefficients for hydrogen, helium, nitrogen, carbon dioxide and air.

These prediction methods apply to any of the problems of predicting friction pressure loss through a passage, from the most simple piece of tube to the complex cooling passages of a regeneratively-cooled rocket nozzle.

Source: M. F. Taylor
Lewis Research Center
(LEW-10774)

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FREQUENCY DOMAIN ANALYSIS AND SYNTHESIS OF LUMPED PARAMETER SYSTEMS USING NONLINEAR LEAST-SQUARES TECHNIQUES

A generalized technique for the design and analysis of linear systems in the frequency domain has been developed that greatly improves the design procedure for filter and compensation circuits. The evaluation of lumped parametric system models has been found to be simplified and computationally advantageous in the frequency domain.

The numerical Fourier transformation equations involved in the frequency domain model are ideally suited for computer calculations. A nonlinear least-squares computer program has been developed which finds the least-squares best estimate for any number of parameters in an arbitrarily complicated model.

Additional advantages of frequency domain modeling include: the facility with which analytical frequency solutions can be obtained as compared with time solutions; there is no change in the form of the frequency solution as the values of the parameters are changed; and the limits of the integral of the squared error can be set to include only the region of interest.

Source: J. R. Hays of
The Boeing Co.
under contract to
Marshall Space Flight Center
(MFS-15033)

Circle 11 on Reader's Service Card.

ESTIMATING RELIABILITY BY APPLICATION OF MATRIX REPRESENTATION

In estimating the reliability of a manned spacecraft for a predetermined mission, there are two probabilities to calculate: (1) mission success—completing the mission objectives with no loss of life, and (2) crew safety—returning the crew safely to earth with or without having accomplished the objectives of the mission. The large number of factors that must be considered when estimating these probabilities, such as the changing hardware configuration and the numerous components in a manned space vehicle, make computations complicated.

A new and simplifying technique for performing this calculation has been developed, based upon matrix collapsing. This technique provides analytic expressions of mission success and crew safety for each subsystem, relating changes in subsystem reliability directly to mission success and crew safety.

The reason for the development of the concepts of matrix representation and matrix collapsing is that ordinary Boolean techniques are cumbersome in cases where the configuration of a complicated subsystem is permitted to change with respect to time. These configuration changes may be due to changing functional or reliability requirements on the subsystem.

The functional and reliability requirements of any given subsystem, "i", for any phase, K, are represented in matrix form in order to facilitate deriving an analytic reliability function for the subsystem which is valid form time zero to the end of that particular phase. Even though there are three mutually exclusive states to consider—mission success, abort enabling failure, and fatal failure—the only two matrices which need to be determined are the success matrix which contains all of the distinct success paths through the subsystem, and the abort enabling failure matrix which contains all of the distinct abort enabling failure paths through the subsystem. The probability of a fatal failure at the end of any phase K is the probability of success at the end of phase K-1 minus the probability of crew safety to the end of phase K, with the total probability of a fatal failure being the sum of all the phase fatal-failure probabilities. The problem of determining the unconditional success matrix can be solved by utilizing the concept of matrix collapsing.

Given two success matrices, A and B, where A is valid for the first mission phase and B is valid for the next mission phase in which the subsystem occurs, every row of A is intersected with every row of B and all redundant paths are eliminated. This process is continued for all subsequent mission phases in which the subsystem occurs; this method is the same as intersecting the Boolean phase models, but is more simple. The abort enabling failure ground rules are applied to the unconditional success matrix to determine the unconditional abort enabling failure matrix. In order to include the aborts, the unconditional success matrix and the unconditional abort enabling failure matrix are collapsed with the subsystem functional and reliability requirements abort matrix. Once the unconditional matrices are determined, a reliability measure may be applied to them; the necessary functions are then derived in order to determine the probabilities of mission success and crew safety.

Source: W. L. Austin of
General Electric Co.
under contract to
NASA Headquarters
(HQN-10246)

Circle 12 on Reader's Service Card.

MATHEMATICAL DETERMINATION OF PARTICULATE CONTAMINATION LEVELS FOR SURFACE CLEANLINESS OF FLUID SYSTEMS

A hypothesis that small particles exist as particulate universes with a geometric mean size of approximately one micron applies in the paint, ceramic, and coal industries and serves as the basis for previous limitations on sizes and quantities of particulate contamination permitted on cleaned surfaces of fluid systems. However, this hypothesis does not apply to complex systems which are unique and have their own requirements for defininining what can be tolerated in fluids. A new method for the determination of particulate contamination levels for use in complex systems requires: (a) the definition of a particle by a mathematical model; (b) a method for calculating the degree of contamination that can be tolerated; and (c) a method for estimating the probability that the contamination occurring on a surface will migrate with the fluid in the system.

(a) Mathematical model: A projected diameter is defined as the diameter of a circle having the same area as the projected image of the particle when viewed through a microscope in the direction perpendicular to the plane of greatest stability. The one-dimensional projected diameter of a

particle is related to a three-dimensional sphere by means of an equation employing a geometrically equivalent shape factor. The particle is subsequently calculated as an equivalent sphere by a conventional mensuration formula.

(b) Contamination tolerance: Operational reliability of a system is determined by the magnitude of contamination deposited at critical clearances of mating parts and by the capability of these parts to continue to function. If a critical clearance is considered equivalent to a single pore, filter media filtration mode equations can be used to derive an equation for correlating the volume of a critical clearance to the number of particles of a specific maximum size that can be tolerated to be deposited at that clearance.

(c) Estimation of probability: Consider each particle an event in that class. The disposition of each particle (event) is mutually exclusive. Either the particle remains on the surface of the cleaned system or the fluid in the system removes it from that surface. Once entrained in the fluid system, the particle poses the threat of being deposited at a critical clearance. Whether the particle "stays" on the surface or "goes" with the fluid determines whether the event is favorable or unfavorable, respectively. Probability equations are used to calculate the probability of an unfavorable outcome for a single trial and the mean number of unfavorable events for "n" trials. Limitations can then be placed on the probability of an unfavorable outcome.

Source: Hayes International Corp.
under contract to
Kennedy Space Center
(KSC-10267)

Circle 13 on Reader's Service Card.

ACOUSTIC WAVE ANALYSIS

A study has been made of the mechanism of generation of acoustic waves in a rotor/stator combination with a comparison of the relative strengths of waves initiated by different sources in a centrifugal pump.

The velocity leaving a blade row is nonuniform due to both the gradients in the region designated as the potential flow and the viscous boundary layers along the blade surfaces, but is periodic in the mean, the period extending from blade to blade. This nonuniform velocity would appear to a downstream blade row in relative motion to the upstream blade row as a periodic unsteady velocity field which produces unsteady pressures on the downstream blades. Acoustic waves are generated by both the unsteady velocity field at the entrance of the blade row, and the unsteady blade pressure loads on the downstream blades due to the unsteady velocity field.

A primary effort was made to indicate the relative importance of the acoustic waves generated by these two related mechanisms. Before this could be done, the amplitude of the unsteady pressures on the downstream blade row had to be calculated. In the literature, these unsteady pressures have been investigated as a function of four effects which have been designated as the circulation, blade thickness, wake, and wake distortion effects. The analytical development of each is based primarily on the two-dimensional theory of the unsteady flow about a thin airfoil. Only the viscous wake effect was adopted for use in the current program, a computer program being written to calculate the unsteady pressures on a blade due to an approaching arbitrary wake velocity.

The generation of waves into a medium surrounding a plate, by unsteady forces on the plate surface, is a coupled problem requiring solution of the equation of motion for transverse displacement of the plate, as well as solution of the wave equation in the medium. The two solutions are coupled by the boundary conditions which require continuity of both the normal velocity and pressure across the fluid-plate interface. To estimate the order of magnitude of the wave amplitude generated by this coupled motion, a simpler example problem was considered which consisted of a plate of infinite extent and uniform thickness. The fluid medium above the plate was assumed to be coupled to the plate motion, but the fluid below the plate was not. The coupled equations were solved, yielding an expression for

the amplitude of the waves generated by an unsteady force on the plate.

To estimate the order of magnitude of the amplitude of the waves generated by an unsteady velocity boundary condition at the inlet of a blade row (or duct), a second simple example was assumed which consisted of a straight, infinite, rectangular duct with a simple harmonic velocity at its inlet boundary, which velocity was uniform in the plane of the duct. The wave equation was solved to yield the generated wave amplitude.

Using data for a standard centrifugal pump, the amplitudes of the oscillations generated by these two mechanisms were computed and compared. This comparison indicated that the amplitudes of the waves generated by the pressure loading on the blades (which in this case were volute tongues) were less by a factor of 10^4 to 10^5 than those generated by the velocity boundary condition. Thus, the primary mechanism for generation of acoustic waves in the centrifugal pump, due to the rotor/stator interaction, is that of an unsteady source at the entrance of the blade row as represented by the unsteady velocity field.

Source: E. D. Jackson of
North American Rockwell Corp.
under contract to
Marshall Space Flight Center
(MFS-18076)

Circle 14 on Reader's Service Card.

ANALYTICAL DRAFTING CURVES PROVIDE EXACT EQUATIONS FOR PLOTTED DATA

A rapid and accurate method has been developed for obtaining explicit mathematical expressions for any numerical data that appear in the form of graphical plots, i.e., thermodynamic properties of gases from a Mollier chart, or physical properties of materials.

Such a method allows a considerable saving in time and effort when large quantities of tabulated data are needed in a particular computing program and there is insufficient storage space in the computer. An almost unlimited amount of data can be accurately represented by an exact mathematical equation and two transformation equations.

Given a set of data points that have been plotted in the coordinate system X_t-Y_t, one or more of the analytical drafting curves can be so oriented as to fit the plotted data to any desired degree of accuracy. In the drafting curves coordinate system X_r-Y_r, there results an exact mathematical expression or set of expressions for the plotted data. By use of the transformation equations that involve translation and, in general, rotation, the plotted data can be represented in the primary coordinate system X_t-Y_t. Therefore, the transformation equations (which are simple relations and are well known in mathematics) plus the mathematical equation of the drafting curve can be used to analytically represent all of the plotted data.

It has been found from actual tests with second degree curves that the aforementioned method can easily provide agreement between the plotted data and the mathematical equations of better than 99% (where the plotted data are accurate). Experimental data, of course, would be within the accuracy with which a smooth curve represented the actual values.

Tabulated data from solutions of high order equations often cannot be conveniently subjected to "curve fitting" by computer. Such data in graphical form, however, can be conveniently represented by the analytical drafting curve method. This method is most conveniently applied when the plotted data are single valued in both coordinate systems. The use of analytical drafting curves has application to the teaching of analytic geometry, since two dimensional coordinate transformations and curves that result from equations with multiple roots can be graphically demonstrated in a unique manner.

Source: R. B. Stewart
Langley Research Center
(LAR-90285)

Circle 15 on Reader's Service Card.

NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS OF VARIOUS ORDERS

Mathematical studies have resulted in techniques for the numerical integration of differential equations of various orders. Modified multistep predictor–corrector methods for general initial-value problems are discussed and new methods are introduced. Preliminary numerical evidence indicates that this process is faster and more accurate than similar methods applied to equivalent systems of first-order equations.

The investigation was oriented towards dealing with higher-order equations directly, rather than converting them to a large system of first-order equations. It was found that the Nordsieck-type methods (N methods) were the most applicable to higher-order equations since the derivatives are directly available. In pursuing the relation between multistep methods and N methods, a large class of modified multistep methods became apparent. These k-step methods can have degree 2k, be strongly stable, and yet only use one derivative evaluation.

An equivalence relation between the methods was determined. The Nordsieck formulation was found to be best for high-accuracy problems, but for low-accuracy problems either the N methods or versions of the new methods should be used. The conventional Adams multistep methods should almost never be used.

The system of equations studied consisted of s equations in the s unknowns y_i, their first p_i derivatives, and the independent variable x. The most general case for which concrete results are given is when these equations can be solved for the highest-order derivatives of each y_i.

For didactic reasons, the first sections of the study deal with the simplest case, $s = p = 1$, in order to prepare for discussions of the advantages of various formulations. Later, the N methods are applied directly to the above equation. A numerical comparison of the results of integrating the second-order equation describing $J_{16}(x)$ directly and integrating the related pair of first-order equations is reported. Evidence indicates both a time and accuracy advantage in the direct approach. Some preliminary tests on the integration of singular families are also reported.

Finally, existence and convergence proofs are presented. The matrix formulation used provides a convergence proof equivalent to the usual proof, but considerably more compact and applicable to the system.

Source: C. W. Gear
University of Illinois
under contract to
Argonne National Laboratory
(ARG-10247)

Circle 16 on Reader's Service Card.

STRUCTURE OF THE ISOTROPIC TRANSPORT OPERATORS IN THREE INDEPENDENT SPACE VARIABLES

The use of elementary functions for construction of solutions for more than one space dimension, if only for special geometries, had been previously attempted. In this work, a decomposition of the transport operator in three space dimensions is carried out.

Based on the idea of separation of variables, a spectral theory for the three-dimensional, stationary, isotropic transport operator is developed in a vector space of complex-valued Borel functions resulting in continuous sets of regular and generalized eigenfunctions.

Because of the non-self-adjoint nature of this operator, the results could not be anticipated intuitively from the known decomposition of this operator in the special case of plane geometry. The results obtained indicate a promising new approach to analytic solution of the linear transport equation in higher space dimensions. Examples are given for slab geometry with and without axial symmetry, spherical symmetry, and cylindrical symmetries.

This method is suggested by the principle of separation of variables, which is widely used for the decomposition of linear partial-differential equations of mathematical physics. The solution

space for the transport equation is mathematically defined as a set of complex valued Borel functions. For an attempt at completeness it was necessary to add to the set of regular eigenfunctions, and the corresponding regular expansion modes over a complex continuum, a set of generalized eigenfunctions and the corresponding generalized expansion modes over a complex continuum of even higher dimension. The regular eigenfunctions are characterized by a complex vector called a regular (or proper) eigenvector, Λ. These Λ eigenvectors, however, must satisfy the condition $\Lambda \cdot \Lambda = \lambda_0^2$, where λ_0^2 assumes exactly one value for any given equation; therefore $\pm\lambda_0$ can be called eigenvalues. For yield of the generalized eigenfunctions the improper integral must be evaluated.

A basic result of this research is the general solution to the transport equation in a sense analogous to that of the theory of differential equations. Special representations are obtained by imposition of geometric restrictions on the general solution. These enable the numerical analyst to construct exact test problems for checking the performance of general multidimensional transport codes, and therefore can be most helpful in error-analysis. The calculation of singular integrals is not difficult if properly executed. The results of the work can be generalized to anisotropic scattering, multiregion, and multigroup problems. A similar theory can be conceived for the discrete ordinate approximations; their spectra, modes, and general solutions can then be compared with the continuous case for a measure of fitness for the approximate solutions.

Source: E. H. Bareiss and I. K. Abu-Shumays
Argonne National Laboratory
(ARG-10448)

Circle 17 on Reader's Service Card.

ANALYSIS OF MAGNETICALLY-CONTROLLED PROCESSES IN PULSE-MODULATION SYSTEMS

Simple analytic expressions have been established which are applicable to the design of a class of pulse modulators in which the modulating signal controls the reset level of flux in a nonlinear magnetic core. Analysis was carried out in the reset-coordinate plane, B_r-T_r, by defining two constraints on the reset process; one imposed by the magnetic core itself, and the other which includes the circuitry external to the core. Expressions derived for modulator response, modulation sensitivity, and limit-cycle stability were applied to both pulse-width and pulse-rate modulator designs.

Basic character of modulator behavior was shown to depend on the relative magnitudes and algebraic signs of the circuit derivative dB_r/dT_r and the core partial derivative $\partial B_r/\partial T_r$. Modulator response was found to be oscillatory for designs in which $(\partial B_r/\partial T_r)/(dB_r/dT_r) < 0$, and is monotonic for $(\partial B_r/\partial T_r)/dB_r/dT_r) > 0$. Modulator stability depends only on the magnitude of $(\partial B_r/\partial T_r)/(dB_r/dT_r)$. Discontinuities in the fundamental mode of operation or transitions to a subharmonic mode were shown to result from the onset of instability. The region of stable operation is extended and the modulator response made very fast for a design using constant reset time.

Results of the investigation show that modulator performance parameters can vary over a wide range in the region of normal operation, with a significant deterioration in response occurring as the threshold of instability is approached. Pulse-width modulator designs with both finite and infinite circuit derivatives were analyzed for operation under constant-voltage reset conditions. Essentially optimum modulator performance was obtained for operation under fixed-reset-time conditions (infinite circuit derivative). The results of the generalized analysis were also applied to a pulse-rate modulator operating under constant-current reset conditions. Both an analytical description of the magnetic core reset function and a graphic representation of an experimentally derived characteristic were used in the analyses, thus illustrating the flexibility of the generalized approach.

Performance parameters of each of the above modulator circuits were measured and compared with the theoretical and calculated values. In each case good agreement was obtained and the results of the generalized analysis thereby substantiated.

Source: Leo J. Veillette
Goddard Space Flight Center
(GSC-10241)

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FINITE ELEMENT FORMULATION FOR LINEAR THERMOVISCOELASTIC MATERIALS

The finite difference equations in time and finite element matrix equations in space have been developed for general linear thermoviscoelastic problems.

There have been numerous applications of the finite element technique, mainly to elastic and plastic static problems and to some steady-state dynamic problems. However, the extension of this technique to viscoelastic problems without using the elastic-viscoelastic correspondence principle has been accomplished only in a few relatively simple cases. It has been concluded that the elastic-viscoelastic correspondence principle can be evoked for rather special cases, i.e., when the material properties are independent of thermal changes. Since the properties of most viscoelastic materials are highly temperature-sensitive, the development of a general program should be based on the solution of integral equations in real time rather than the correspondence principle.

A brief statement of the thermoviscoelastic field equations is followed by the development of the finite different equations in time and then by the finite element formulation in space. The equations are derived for a general three-dimensional body but are applicable with minor changes to one- and two-dimensional configurations. Some attention is given to the experimental determination of material properties and their use in analytical work. An expansion of the experimentally or analytically determined material property functions in terms of exponential series leads to recurrence matrix equations, eliminating the problem of calculating at each time-step the history of material response.

Source: E. Heer and J. C. Chen of
Caltech/JPL
under contract to
NASA Pasadena Office
(NPO-11229)

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SOME NUMERICAL METHODS FOR INTEGRATING SYSTEMS OF FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS

Numerical methods for integration of systems of coupled first-order ordinary differential equations have been studied and developed. The extrapolation methods of Burlirsch-Stoer and Neville are explained, and the results of tests comparing these extrapolation methods with the Runge-Kutta and Adams-Moulton methods are given. There are some circumstances under which the extrapolation method may be preferred. Results of double-precision tests of the Bulirsch-Stoer method show the efficiency of various orders of this method as a function of desired accuracy.

The speed and accuracy of the methods were compared by programming the methods on the same computer (CDC-3600) and applying each to the same sets of equations. The numbers of times that the functions to be integrated had to be evaluated, for a desired accuracy, were used as a computer-independent and program-independent measure of time. The accuracies were compared by use of each method to integrate from given initial conditions to some final point at which the relative error was computed.

The results of cases run by the four methods tested indicate that the efficiency of the solution is dependent on proper matching of the order of the method of solution to the desired accuracy. For example, in tests on the negative-exponential

equation, for a relative error of 10^{-16}, the Neville method with M = 4 takes twice as long as the method with M = 6 and about 2.5 times as long as the shortest time with M = 10. The Runge-Kutta fourth-order method and the Adams-Moulton fourth-order method take about 160 times as long as the Neville method with M = 10, or about 250 times as long as the Bulirsch-Stoer method with M = 10. Even at relative errors of 10^{-9}, the Adams-Moulton and Runge-Kutta methods take about 9 times as long as the Neville and Bulirsch-Stoer methods with M = 6. At the other extreme, if a rough estimate is desired and errors up to 10^{-2} can be tolerated, Neville's method with M = 10 takes almost 7 times as long as the fourth-order Adams-Moulton methods, and the methods of lower order are preferable.

The double-precision tests of the Bulirsch-Stoer method show it to be quite powerful for work of great accuracy; it seemed especially capable of coping with changing difficulty of solution of the equations presented to it.

Source: N. W. Clark
Argonne National Laboratory
(ARG-10308)

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CONTROLLABILITY OF DISTRIBUTED-PARAMETER SYSTEMS

The controllability of distributed-parameter control systems (those which can be described by partial differential equations) has been studied, resulting in a general theory to include those control systems which cannot be described by ordinary differential equations.

The study summarizes techniques applicable to control systems problems. The eigenvalue-eigenfunction expansion method for the solution of homogenous boundary value problems (BVP) is used. Problems in which the control appears at the boundary are treated by converting the nonhomogenous BVP to an equivalent homogenous BVP by introducing generalized functions. The generalization of the concept of controllability of finite dimensional systems to infinite dimensional systems is given. The pseudo-inverse of a linear operator is defined which is a generalization of that of a matrix for finite dimensional spaces. The pseudo-inverse is then used to obtain minimum energy control for distributed-parameter systems. This generalization includes results for finite dimensional systems which are available. In the infinite dimensional problem, it is necessary to solve for the eigenvalues and eigenfunctions of an integral operator. The necessary and sufficient conditions for the states which are reachable when the control is required to satisfy a norm constraint are given. The conditions are obtained by an application of the moment problem to distributed-parameter systems. These results are then used to obtain conditions for complete controllability.

Source: C. J. Herget of
University of California
under contract to
Marshall Space Flight Center
(MFS-19429)

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LINEAR SYSTEMS OF EQUATIONS SOLVED USING MATHEMATICAL ALGORITHMS

A new mathematical algorithm has been developed for the solution of linear systems of equations, AX = B, which preserves the integer properties of the coefficients. The algorithms presented can also be used for the efficient evaluation of determinates and their leading minors.

The methods described are computationally more efficient than the corresponding single-step Gaussian elimination technique currently being used. In addition, the magnitude of the coefficients in the transformed matrices is minimized. The algorithms have universal applicability for solving linear equations and are particularly adaptable to digital computer techniques.

Source: E. H. Bareiss
Argonne National Laboratory
(ARG-10146)

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SHOCK AND VIBRATION RESPONSE OF MULTISTAGE STRUCTURE

An analytical and experimental study has been made on the shock and vibration response of a multistage structure. Lumped-mass, continuous-beam, multimode, and matrix-iteration methods were used in the analytical phase of the study. Experimentally, a special technique was used in conjunction with a mechanical shaker. The study was made on the load paths, transmissibility, and attenuation properties along a longitudinal axis of a long, slender type of structure with increasing degree of complexity including ring frames, longerons, bulkheads, propellant fluid, payload mass (spacecraft), as well as multistage structures. Free vibration characteristics were analytically solved by lump masses and continuous beam approaches. A modal approach was applied to determine shock responses to various phases of different forms and durations for a multidegree-of-freedom system. In the experimental phase of the study, a special technique was employed to produce pulses of varying durations by a mechanical shaker. The test results verified the analytical predictions to a satisfactory degree.

Longitudinal forced response to various pulses of different forms and durations can be calculated easily for slender or multistage structures, as in bridge support towers, and multistory building supports. Solutions obtained are peak responses and complete shock wave propagation along the axial stations within the structure.

Source: S. Y. Lee, S. S. Tang, and J. G. Liyeos of
North American Rockwell Corp.
under contract to
Marshall Space Flight Center
(MFS-14972)

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AN ORTHONORMALIZATION PROCEDURE FOR MULTIVARIABLE FUNCTION APPROXIMATION

In many types of scientific and engineering problems, a table of two or more columns of data occurs and it is often desirable to present this data in a more useful form. The usual methods for performing this task are the many different techniques of multivariable function approximation such as the least-squares procedures that require appreciable time to compute the coefficients.

Where a function of several variables is given numerically in tabular form, an orthonormalization technique allows an approximation of the numerical data to be determined in a convenient functional form. The method requires much less computational work than the usual least-squares technique, and allows more easily controlled accuracy. In this technique, the speed and accuracy of coefficient computation are much improved. Additionally, a very clear and useful physical interpretation of the procedure is available to aid in the choice of terms to be included in the approximating formulas.

Source: H. L. Ingram
Marshall Space Flight Center
(MFS-1313)

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MINIMUM PERMISSIBLE LEAKAGE RESISTANCE ESTABLISHED FOR INSTRUMENTATION SYSTEMS

When an instrumentation system has been exposed to the elements, its leakage resistance to ground is appreciably affected to the detriment of system precision. Previously, such exposed systems have been dried out until leakage resistance to ground approaches infinity. This often far exceeds the precision/frequency requirements of a given system and thus results in a waste of time and money.

Mathematical formulas are used to determine if, and to what extent, a given system should be dried out to restore minimum permissible leakage resistance.

One example is a system that uses transducers

associated with a strain gage bridge network. The power supply minus lead of the bridge network is grounded at the test stand. Leakage from one of the bridge output leads to the stand ground unbalances the bridge. If the amount of leakage during a test changes sufficiently, the test data are invalid. Invalid data are readily identified by a significant zero shift and significant difference between post-test and pretest electrical unbalance.

In a pressure measuring system that measures one megohm leakage to ground, system precision must not deteriorate by as much as 20%, i.e., 0.0005 (0.2 × 0.0025). To determine whether the system should be dried out or the test run without this precaution, the following formula is employed:

$$\Delta R = \frac{(10^6)^2\Omega}{10^6 + \frac{0.8(35{,}000)}{0.0005}}$$

$$\Delta R = 17{,}550\Omega$$

where: ΔR = Change in leakage resistance during test.

This example shows that if the leakage resistance increased or decreased 17,550 ohms from the initial value of 1 megohm during test, system precision would deteriorate 0.05%. Because 17,550 ohms represents only a 1.755% change, it is likely that the leakage resistance would change by this amount. In this case, the system would be dried out before running the test.

Assuming the system dried out so that leakage to ground measures 10 megohms, the question of whether to dry out the system further or run the test is resolved by the following:

$$\Delta R = \frac{(10^7)^2\Omega}{10^7 + \frac{0.8(35{,}000)}{0.0005}}$$

$\Delta R = 1.5M\Omega$ or 15% change.

In this case the test would be run without further drying since it is unlikely the leakage resistance would change as much as 15% during test.

When it is assumed the leakage resistance will not change more than 20% during a test, and that a deterioration in precision of 20% of the precision classification of the system can be tolerated, the following formula determines the minimum permissible leakage resistance:

$$R_L = \frac{R_C}{NP}$$

where: R_L = Minimum permissible leakage resistance; R_C = Resistance of electrical unbalance resistor for 80% deflection; N = Number of electrical unbalance resistors; P = System precision classification (0.0025, 0.005, 0.01, 0.02, etc.).

Formulas may be derived to be used for an indeterminate number of instrumentation systems that are exposed to moisture penetration.

Source: J. L. Perrin of
North American Rockwell Corp.
under contract to
Marshall Space Flight Center
(MFS-848)

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Section 2. Computer Programs for Industrial Applications

COMPUTER PROGRAM (P1-GAS) CALCULATES P-0 AND P-1 TRANSFER MATRICES FOR NEUTRON MODERATION IN MONATOMIC GAS

To generate multigroup thermal neutron scattering cross sections for use in neutron transport theory programs, it is necessary to have the group-to-group elastic transfer kernels. Below 1 eV, experimental crystalline transfer kernels may be used, at about 1 eV, it is necessary to analytically generate the elastic transfer kernels. The problem is to generate the P-0 and P-1 transfer matrices for neutron moderation in a monatomic gas.

The equations programmed in the P1-GAS FORTRAN IV Program are essentially those derived by Clendenin and others. These equations for the elastic scattering kernels are based on the following conditions: (1) there is isotropic scattering in the center-of-mass coordinate system; (2) the scattering cross section is constant, i.e., independent of the velocities; and (3) the target nuclear velocities satisfy a Maxwellian distribu-

tion. The monatomic gas model has the advantage that the differential cross section (matrix elements) may be expressed analytically.

To calculate the elastic scattering transfer matrices for a neutron in a solid moderator, one of three models may be used, depending on the energy of the neutron and the temperature of the moderator. When the neutron energy is greater than a few eV, the scattering process depends only weakly on the facts that the target nuclei are bound in the lattice and the target nuclei have thermal motion. Thus, the moderator nuclei may be treated as free and stationary. In this model the neutron energy cannot increase as a result of a collision. For lower neutron energies (the thermal range) "upscatter" can occur due to the thermal motion of the moderator nuclei. For neutron energies less than about 1 eV, the interaction of the neutron range (energies above about 1 eV), the gas model described here is applicable. In the gas model, no account is taken of the fact that the scattering nuclei are bound in ordered fashion in the lattice; however, the thermal motion of the target nuclei is considered.

When neutrons are moderating in a monatomic gas such as helium, the results of P1-GAS are directly applicable. In this case the thermodynamic temperature of the gas characterizes the Maxwellian distribution. However, for a molecular gas such as hydrogen, the results in general do not apply since the internal degrees of freedom (vibrations and rotations) are not treated in the gas model.

Quantities are printed out in the program that provide a check of the calculation to ascertain whether or not specific conditions are satisfied by the transfer matrices.

This program is written in FORTRAN IV and MAP for the IBM 7090/7094 computer.

One of the input quantities to the P1-GAS program is the temperature of the gas. Lamb first showed (for the resonance absorption of neutrons) that a Maxwellian distribution may be used for the nuclear velocities in a Debye solid. Nelkin and Parks showed this was also true for slow neutron scattering. However, the Maxwellian distribution is not characterized by the thermodynamic temperature T^1 of the moderator, but an effective temperature T^1 directly. Actually, there are additional methods for obtaining T^1.

Source: G. Gibson and G. Collier of
Westinghouse Astronuclear Laboratory
under contract to
Space Nuclear Propulsion Office
(NUC-10141)

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MOP (MATRIX OPERATION PROGRAMS SYSTEM)

The programming system consists of a set of FORTRAN IV subroutines which are related through a small common allocation. The system accomplishes all matrix algebra operations plus related input-output and housekeeping details. It was coded for the IBM 7094 DCOS, but the document contains specific instructions for conversion to other computers. It is a convenient and complete programming base for jobs using substantial matrix algebra manipulations, and in no way prevents the user from calling other subroutines.

The system can be used by writing a main program which calls the MOP system subroutines. After the system is initialized, all matrix algebra operations can be performed. The casual programmer can operate in an automatic nominal mode; the professional programmer has access to all system control flags and may exercise subtle options within the system. The naming of calling sequences and operations is done according to an easily mastered mnemonic scheme.

A simple overlay scheme allows loading of the entire system in 6000 words of core, which, with the system subroutines and a substantial main program, leaves sufficient space for two 50 × 50 square matrices with four symmetric operators overlaid for flexibility. The sophisticated housekeeping and internal conversions of the system may be loaded if desired, but high efficiency is not claimed for programs desiring only a few calls.

The above applies to double precision word length, but the system can be rapidly converted to single-precision if desired.

This system is written in FORTRAN IV for use on the IBM 7094 computer. Timing and program accuracy performance is good, and complete debugging programs are included which automatically test the system. The checkout has been exhaustive, and the document is quite complete.

Source: P. M. Muller
NASA Pasadena Office
(NPO-10429)

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COMPUTER PROGRAM FOR MASS OPTIONAL SOLUTIONS OF SOME END-POINT TRAJECTORY PROBLEMS

The calculation of trajectories is required for a variety of orbital problems such as orbit transfer, rendezvous, lunar transfer, and lunar launch. In all cases optimization of trajectories for minimal propellant consumption is a prime concern. This program incorporates the technique of calculating coast arcs into a three-dimensional fixed end-point steepest ascent computer program. There is no restriction on the magnitude of thrust or on initial or final orbit characteristics. The initial conditions and desired terminal conditions of a transfer trajectory can be specified in conventional orbital elements or in spherical coordinates.

The equations of motion are evaluated in spherical coordinates whatever the form of the input. Hence, the program actually calculates a trajectory between any two points in space defined by initial and final position vectors. The corresponding end-point velocities are equally arbitrary. Problems can be handled which, on the surface, do not appear to fall within the orbit transfer category.

The problem of thrusting flight in a vacuum in the presence of two-body forces only is formulated in the calculus of variations and solved by the method of steepest ascent. Optimal thrust directions and thrust durations are found for a variety of orbital transfer problems in two or three dimensions. Constant thrust and specific impulse are assumed.

The program can be used on either the IBM 7094 or SRU 1107 and has been written in FORTRAN IV language.

Aerospace, missiles, ballistics, physics, and geology have some general similarity in flight problems, either of projectiles or particles, to which this program is applicable. Also this technique could be of value in other isolation techniques such as the calculus of variation or Newton-Raphson.

Source: M. S. O'Mahony, A. G. Bennett,
and C. D. Eshridge of
The Boeing Company
under contract to
Marshall Space Flight Center
(MFS-12976)

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HICOV (NEWTON-RAPHSON CALCULUS OF VARIATION WITH AUTOMATIC TRANSVERSALITIES)

In generating trajectories that are optimum with respect to payload placed in an earth orbit, there is a need for analytically forming the transversality equations and their respective partial derivatives.

A computer program generates the desired trajectories through the use of a subroutine package which produces the terminal and transversality conditions and their partial derivatives.

The Wierstrass condition defines the control variables, or steering angles, which appear in the differential equations of motion, while the transversality conditions are used along with the physical cutoff conditions to determine the nine boundary conditions. The partial derivatives are used to solve for the desired boundary conditions and then determine the gain matrices for the steering angles. The gain matrices are used to write the guidance functions which reduce to linear

polynomials in the state variables. The state variables can be stored onboard the vehicle for guidance purposes.

The transversality generator program along with the FORMAC Subroutine package, TAC, generates the terminal and transversality conditions and their partial derivatives. The package uses a formula manipulation language which is an extension of FORTRAN. It allows the programmer to arrive at a single analytic solution, following it by multiple numeric evaluations.

The terminal conditions are input into the program and a FORTRAN IV source deck for the calculation of the solutions is generated on tape. Included in the tape source deck are a deck name, calling sequence, type statements, dimension statements, and analytic expressions as desired. The tape can then be punched on cards and used directly with a program or called in by a program through the use of $ IEDIT.

Using this subroutine package, the programmer is able to automatically arrange his analytic solution(s) in form suitable for numeric evaluation.

This program is written in FORTRAN IV and FORMAC for the IBM 7094 computer.

Source: T. J. Heintschel of
General Electric Company
under contract to
Marshall Space Flight Center
(MFS-14468)

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LEAST-SQUARES SMOOTHING (LSSMG) PROGRAM

This program was developed so that the smallest computation time could be used to obtain a "smoothed" value for 21, 41, 61, or 81 points of raw data. For a slightly longer time, any odd numbers of points ≤ 81 may be used.

For an odd-numbered set of ordered pairs (x, y), which should represent some approximation to a function $y = f(x)$ which is at most second degree, LSSMG will automatically provide the medium pair (x_m, y_m) and will also provide the derivative dy/dx at the point $x = x_m$, computing so much of the function $y = a_1 + a_2x + a_3x^2$

as is required to give a_1 and a_2

$(= dy/dx$ when $x_m = 0)$

Since the primary purpose is speed, the function is not computed, nor is the array of smoothed points computed, but these may be obtained by the calling program by means of additional FORTRAN coding. An error routine is included for $n > 81$ or n even.

Language: GMAP Assembly Language

Machine Requirements: GE-625

Source: Wallops Station
(WLP-10031)

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CALCULATION OF THE VOLUME OF CAVITY ON AN INDUCER BLADE, ASSUMING A FINITE LEADING-EDGE CAVITY

This program calculates cavity boundaries, such as the shape and cavity volume, for a cavitating inducer blade by evaluating the Free Streamline-Wake Theory. The complex variable potential-flow solution is modified so that the branch cuts needed for evaluation of the solution exactly match the branch cuts defining the Fortran H complex logarithm routine CLOG. The ratio of discharge velocity to the inlet velocity is expressed as a function of cavitation number K and blade geom-

etry, so that the solution may be evaluated for a series of K values and flow and blade angles. The volume of the cavity is found by integration of the net area between cavity boundary and blade-suction surface.

Output consists of plots of the free streamlines. Also, the value of cavity area, metal area, net cavity length/spacing ratio, and cavity height/spacing are tabulated.

Language: FORTRAN H

Machine Requirements: IBM 360, Release 11

Source: North American Rockwell Corp. under contract to Marshall Space Flight Center (MFS-18093)

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BELLOWS CALCULATION PROGRAM

This program employs empirical and analytical derived design equations on various metal bellows of different sizes in order to calculate various properties of bellows used in ducting systems. Arithmetic operations are performed in double precision. Calculations are restricted to four single bellows movements and two double bellows movements. One subroutine and one data deck are required with the main program.

The main program and the subroutine calculate bellows spring rates, bulging, bending, and hoop stresses. Cycle life is calculated by data deck. With known bellows dimensions and type of movements supplied as data, main program and subroutine calculate spring rate, actuating force, squirming pressure, stress, bellows weight, resonant frequency, fatigue life, and convolution clearing.

Language: FORTRAN H

Machine Requirements: IBM 360, Release 11

Source: North American Rockwell Corp. under contract to Marshall Space Flight Center (MFS-12641)

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CALCULATION OF ISOTHERMAL, TURBULENT-JET MIXING OF TWO GASES

This program solves a simplified model of the turbulent mixing that occurs between a central fuel jet and a surrounding, faster-moving coaxial stream of propellant. The von Mises transformation was used to convert the axisymmetric forms of the isothermal boundary layer momentum and diffusion equations to forms amenable to numerical solution. The effects of confining walls were not considered. The program can solve problems in which the initial velocities and densities of the two streams differ greatly, by using expressions for eddy viscosity that vary radially as well as axially.

The input to the program consists of the initial ratio of coaxial-stream velocity to jet velocity, the mass fraction of component one in the initial jet and in the initial coaxial stream, the ratio of molecular weight of component two to the molecular weight of component one, the constants in the eddy viscosity formulation, the turbulent Schmidt number, the ratio of reactor diameter to jet radius, the constants in the reference density formulation, and the axial positions at which output is desired.

At each axial position, the program prints out the axial position, the eddy viscosity, the product of density and eddy viscosity, and (for axial velocity, mass fraction, and mole fraction) the quantity:

$$\frac{\text{(Centerline value-Coaxial-stream value)}}{\text{(1-Coaxial-stream value).}}$$

The radial variations with stream function, also converted to radial position, and the ratio of radial position to half radius for axial velocity, mass fraction, and mole fraction are provided in the following form:

$$\frac{\text{(Local value-Ambient stream value)}}{\text{(Centerline value-Ambient stream value)}}$$

Also listed are momentum flux normalized with the square of the centerline velocity, mass flux normalized with the product of centerline velocity and mass fraction, and eddy viscosity and the product of density and eddy viscosity divided by their centerline values. Both sides of the centerline compatibility condition are printed next, followed by the values of velocity and density at the largest stream function position in the calculation. Finally, the "line-of-sight" concentration, the dimensionless mass of component one, and the ratio of mass of component one-to-initial mass are listed.

Language: FORTRAN IV

Machine Requirements: IBM 7094

Source: Lewis Research Center (LEW-10579)

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